MicroBooNE Experiment:
Wire Deflection Evaluation

Order:	N/A
Document Reference:	RAP.2008.12.00.ver.1
Issue Date:	12/5/2008
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1 Introduction

MicroBooNE is an approved experiment at Fermilab to build a large liquid Argon Time Projection Chamber (LArTPC) to be exposed to the Booster neutrino beam and the NuMI beam at Fermilab. The experiment will address the low energy excess observed by the MiniBooNE experiment, measure low energy neutrino cross sections, and serve as the necessary next step in a phased program towards massive Liquid Argon TPC detectors.

MicroBooNE liquid argon time projection chamber consists of a rectangular grid of wires installed in a cylindrical argon vessel. Heat through the insulation will cause natural convection up around the vessel shell and down in the middle. The liquid velocity varies depending on location and direction of flow. This analysis should evaluate the effect of flow on the wires:

How much will the wires deflect due to drag (static analysis)?
How much will the wires deflect due to vortex shedding (dynamic analysis)?
How sensitive are these deflections to wire tension?

1.1 General parameters for the study

Wire length: $l = 2.5\ m$.
Wire diameter: $D = 150x10^{-6}\ m$ (cross section A_0)
Wire density: $\rho_w = 7800\ kg/m^3$.
Wire Tension: $T = 10\ N$.
Liquid argon density: $\rho_l = 1395\ kg/m^3$ at saturation point.
Liquid argon dynamic viscosity: $\mu = 259.79x10^{-6}\ Pa.s$ at saturation point.
Maximum transverse liquid argon velocity on wire: $V = 1.25x10^{-2}\ m/s$ *(fig.1)*

2 Static Analysis

2.1 Catenary

It is common knowledge that uniform density cables tensioned in a uniform gravity field (y-direction) hang in the form of a catenary, with vertical deflection:

$$y(x) = \frac{-T}{w}\left[\cosh\left(\frac{w.l}{2T}\right) - \cosh\left[\frac{w}{T}\left(\frac{l}{2} - x\right)\right]\right] \quad (1)$$

Here T is the horizontal (x-direction) component of tension which is uniform from one end of the wire of length l to the other. The weight of the wire per unit length w is also assumed uniform along the wire.

For highly tensioned wires, substitution of the first 2 terms of the power series of the *cosh* terms gives a simple parabolic approximation:

$$y(x) \approx \frac{w}{(2T)}(x^2 - x.l) \quad (2) \text{ and } \quad y_{max} = y\left(\frac{l}{2}\right) = \frac{-w.l^2}{(8T)} \quad (3)$$

2.2 Elastic Catenary

The catenary model assumes that the wire or chain can not transmit bending or torsional moments. It is a structure made up of inextensible links joined with complete angular flexibility. The catenary formula depends only on tension and weight/length. Neither elastic modulus nor wire diameter appears in the equations, no elastic energy is stored.

For real wires, stretch and bending stiffness modify the catenary form, even for thin wires. The case of the stretchable elastic catenary is covered by Irvine [1]. The main effect of stretching is a reduction of the weight/length thus reducing the wire deflection.

Stress across the wire section area A_0 is $\sigma = T/A_0$ and the wire strain is $\varepsilon=\Delta l/l = T/(E.A_0)$ (wire elastic modulus E). When tensioned, wire stretches by $\varepsilon * l$ reducing the weight/length by $1/(1+\varepsilon)$. The parabolic approximation for the mid span sag of a stretchable wire then becomes:

$$y_{max}=y(\frac{l}{2})=-w\frac{l^2}{(8\text{T})}.(\frac{1}{1+\varepsilon}) \quad (4)$$

At first order, y_{max} varies like $1/T$ (ε contributes to $1/T^2$). ε being generally small (less than a percent), elastic catenary modelisation yields results similar to the pure catenary model.

2.3 Wires of the MicroBooNE Time projection Chamber

The wires are in a vertical plane (y-direction), so gravity does not play a role on their deflection, and subjected to drag forces (x-direction) generated by the surrounding liquid argon convection movements.

The general form of the drag force F_D (per unit length l) generated by a fluid of density ρ_l and velocity V, on a wire of diameter D is:

$$F_D=\frac{1}{2}\rho_l D V^2 C_D \quad (5)$$

C_D being the drag coefficient for a cylinder and depends upon the Reynold number (Re) of the fluid.

Substituting w by F_D in equation (2) and inverting x and y axis gives the deflection of a stretched wire subjected to a uniform perpendicular drag force:

$$x(y)\approx\frac{F_D}{(2\text{T})}(-y^2+y.l) \quad (6) \text{ and } \quad x_{max}=x(\frac{l}{2})=F_D\frac{l^2}{(8\text{T})} \quad (7)$$

2.4 Wire Static Deflection Calculations

C_D can be evaluated using [2] and fig. 2. Reynolds number in these conditions is $Re \approx 10$, then:

$C_D \approx 3$

thus:

$x_{max} \approx 4\ \mu m$ and x_{max} varies like $1/T$

3 Dynamic Analysis

The following analysis is presented for vortex shedding from a circular cylinder. If the frequency at which the vortices are shed is near the natural frequencies of the stretched wire then large amplitude vibrations exist.

The first step is to characterize the dynamics of the wire, then vortex shedding frequency range to finally evaluate the forced response of the system.

Effect of liquid argon viscous damping is finally addressed.

3.1 Normal Modes of an Undamped Stretched Wire

Considering only the transverse x-dimension, Newton's second law of motion takes the form:

$$A_0\rho_w\frac{\delta^2 x(y,t)}{\delta t^2}=T\frac{\delta^2 x(y,t)}{\delta y^2}+F_D \quad (8)$$

The classical resolution leads to the normal modes frequencies:

$$f_n = \frac{n}{(2l)} \sqrt{\frac{T}{\rho_w A_0}}$$ (9) The fundamental frequency ($n=1$) is $f_1 \approx 54$ Hz.

3.2 Frequency of Vortex Shedding Excitation from a Circular Cylinder

The frequency of vortex shedding excitation is given by:

$$f_{vs} = \frac{V S_t}{D}$$ (10) St being the Strouhal Number.

St is a function of the Reynolds number [3] and is in the range [0; 0.3] (fig. 3). Below $Re \approx 100$, it is usually admitted that no vortex shedding will develop thus the flow is completely laminar and the static drag applies (in our case $Re \approx 10$).

For being conservative, i.e. vortex shedding developing, let $St = 0.2$ then $f_{vs} = 17\ Hz$ – below the fundamental frequency (54 Hz) of the wire.

The required fluid velocity to get f_{VS} close to the fundamental wire frequency is about 4×10^{-2} m/s (a factor 3 more). Nevertheless, at this velocity $Re \approx 30$ and still no vortex shedding should develop.

3.3 Forced Response of an Undamped Wire Subject to Vortex Shedding Harmonic Excitation

Vortex shedding harmonic excitation, per unit length, is $F_{VS} = F_D \sin(\omega_{vs} t)$. Taking into account a one transverse degree of freedom model, the forced response equation is:

$$\frac{\delta^2 x(t)}{\delta t^2} + \omega_1^2 x(t) = \frac{F_D}{\rho_w A_0} \sin(\omega_{vs} t) \quad (11)$$

with $\omega_1^2 = \frac{\pi^2}{l^2} \frac{T}{\rho_w A_0}$ from equation (9)

and, using equation (5) and (10): $F_D = \rho_l \frac{D^3 C_D}{8\pi^2 St^2} \omega_{vs}^2$ (14)

Substituting equation (14) into equation (11), one obtains:

$$\frac{\delta^2 x(t)}{\delta t^2} + \omega_1^2 x(t) = \frac{\rho_l}{\rho_w} \frac{D^3}{8\pi^2 A_0} \frac{C_D}{St^2} \omega_{vs}^2 \sin(\omega_{vs} t) \quad (15)$$

The harmonic excitation is of “frequency-squared” type, general steady-state solution can be derived [4] and is of the form:

$$x(t) = X \sin(\omega t + \varphi) \quad (16)$$

with

$$X = \frac{\rho_l}{\rho_w} \frac{D^3}{8\pi^2 A_0} \frac{C_D}{St^2} \left[\frac{r^2}{1-r^2}\right]$$ (17) and $$r^2 = \left[\frac{\omega_{vs}}{\omega_1}\right]^2 = \frac{41^2 \rho_w A_0 V^2 St^2}{T D^2}$$ (18)

3.4 Wire Dynamic Deflection and Sensitivity

Rewriting equation (17) using equation (18) to show the St^2 dependency of the amplitude X:

$$X=\frac{\rho_l}{\rho_w}\frac{D^3 C_D}{8\pi^2 A_0}\left[\frac{4l^2\rho_w A_0 V^2}{T D^2-4l^2\rho_w A_0 V^2 St^2}\right] \quad (19)$$

and considering that $C_D \approx 3$ and St is in the range [0; 0,3], one obtains the bounding values for X:

X ≈ [3; 4] μm

X varies like (1/T), augmenting the wire tension T reduces proportionally the dynamic deflection.

3.5 Effect of Liquid Argon Viscous Damping

The effect of viscous damping is twofold: rsing the frequency ratio at which the steady-state amplitude is maximum (r_{max}) and reducing the steady-state amplitude X.

For the frequency ratio, the following equation will hold:

$r_{max}=\frac{1}{\sqrt{1-2\zeta^2}}$ with $\zeta=\frac{\gamma}{2\omega_1}$ the damping ratio and $\gamma=\frac{F_D}{A_0\rho_w V}$ the effective damping [s^{-1}].

Taking the same parameters, one obtains:

$\gamma = 28.5\ s^{-1}$, $\zeta = 4x10^{-2}$ *and* $r_{max} = 1.002$ meaning that viscous damping is negligible.

For the steady-state amplitude, the term $\frac{r^2}{(1-r^2)}$ in equation (17) is replaced by :

$\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ which is 99.96% of $\frac{r^2}{(1-r^2)}$.

4 Conclusion

Analytical expressions of the deflection of a wire subjected to viscous drag have been presented in both static and dynamic situations.

For the static case, the maximum wire deflection is 4 μm and varies like 1/T – T tension in the wire.

For the dynamic case and taking into account the expected maximum transverse fluid velocity on the wire, no dynamic effect should develop. Nevertheless, a conservative value of the dynamic deflection has been evaluated in the range [3; 4] μm, varying also like 1/T. Viscous damping for the dynamic case is negligible.

5 References

[1] H. Max Irvine, "Cable Structures", MIT Press, 1981
[2] H. Schlichting, "Boundary Layer Theory", 7th ed. New York: McGraw-Hill, 1979
[3] F.M. White, "Fluid Mechanics", 4th ed., WCB McGraw-Hill, Boston, 1999
[4] K.S. Graham, "Fundamentals of Mechanical Vibrations", 2nd ed., McGraw-Hill, 2000

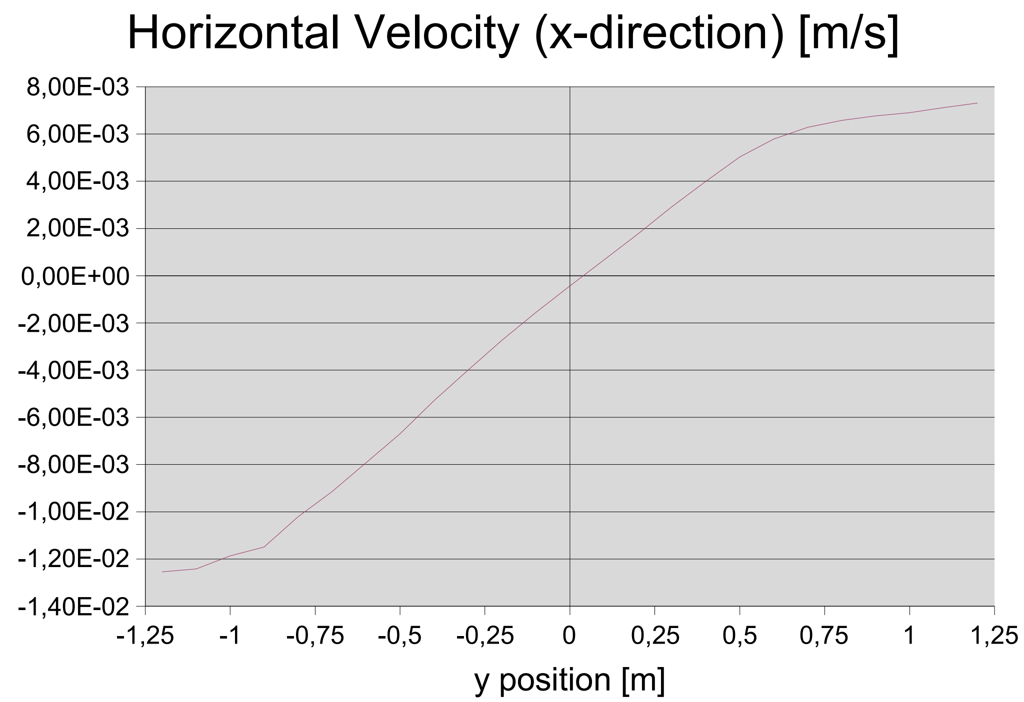

Figure 1: Horizontal velocity of liquid argon along the outermost wire of the time projection chamber (courtesy R. Schmitt)

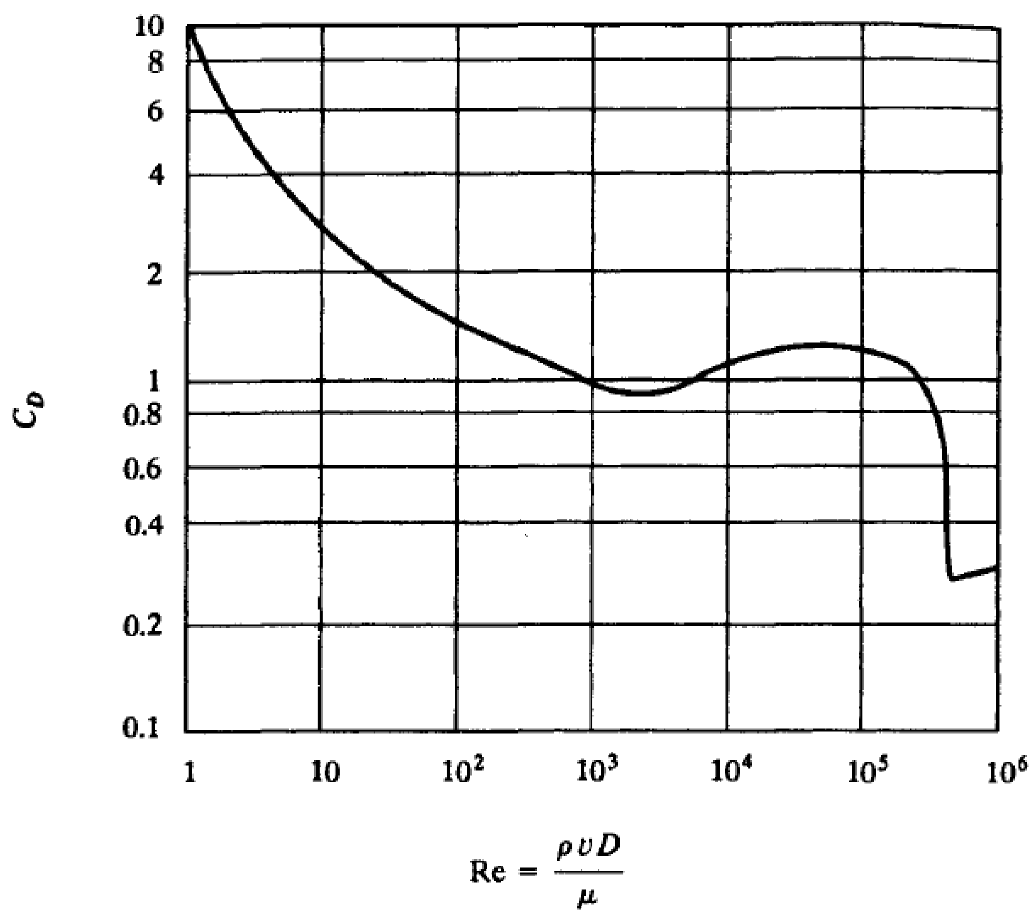

Figure 2: Average drag coefficient C_D for cross-flow over a smooth circular cylinder (from [2])

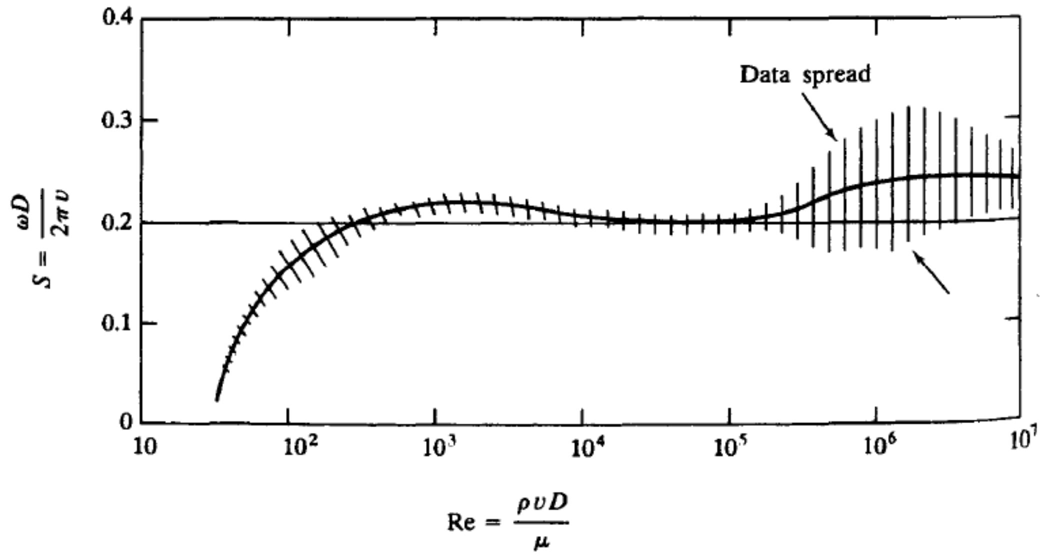

Figure 3: Strouhal number (from [3])